# Economics Lecture 4 

## 2016-17

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## Course Outline

## 1 Consumer theory and its applications

1.1 Preferences and utility
1.2 Utility maximization and uncompensated demand
1.3 Expenditure minimization and compensated demand
1.4 Price changes and welfare
1.5 Labour supply, taxes and benefits
1.6 Saving and borrowing

## 2 Firms, costs and profit maximization

2.1 Firms and costs
2.2 Profit maximization and costs for a price taking firm
3. Industrial organization
3.1 Perfect competition and monopoly
3.2 Oligopoly and games

### 1.3 Expenditure minimization and compensated demand

### 1.3 Expenditure minimization and compensated demand

1. Definitions of compensated \& uncompensated demand
2. Definition of the expenditure function
3. Homogeneity of the compensated demand and expenditure functions
4. Income \& substitution effects
5. The slope of compensated demand curves
6. Compensated demand \& the expenditure function with Cobb-Douglas utility

## Expenditure minimization and compensated demand

7. Compensated demand \& the expenditure function with perfect complements and perfect substitutes utility
8. Properties of the expenditure function
9. The Slutsky equation

## Expenditure minimization and compensated demand

Why bother?

1. Income and substitution effects, essential for understanding the effects of changes in wages and taxes on labour supply and interest rates on savings.
2. Expenditure function, essential for consumer surplus and welfare economics.

# Definitions of compensated and uncompensated demand 

## 1. Definitions of compensated and uncompensated demand

What we have been calling demand up to now is uncompensated (Marshallian) demand which maximizes utility $u$ given prices $p_{1}$ and $p_{2}$ and income $m$, so is a function of $p_{1}, p_{2}, m$, notation $\mathrm{x}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{~m}\right), \mathrm{x}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{~m}\right)$.

Compensated (Hicksian) demand minimizes the cost of obtaining utility $u$ at prices $p_{1}$ and $p_{2}$ and is a function of utility $u, p_{1}, p_{2}$, notation $h_{1}\left(p_{1}, p_{2}, u\right), h_{2}\left(p_{1}, p_{2}, u\right)$.
$x_{2}$
To get uncompensated demand fix income and prices which fixes the budget line.

Get onto highest possible indifference curve.

To get compensated demand fix utility and prices which fixes the indifference curve and gradient of budget line.
Get onto lowest possible budget line.
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## Definition of the

## expenditure function <br> and intuition

## 2. Definition of the expenditure function

The expenditure function $E\left(p_{1}, p_{2}, u\right)$ is the minimum amount of money you have to spend to get utility $u$ with prices $p_{1}$ and $p_{2}$. It is a function of $p_{1}, p_{2}$ and $u$.

The amount of goods which minimizes the cost of getting utility $u$ is compensated demand $h_{1}\left(p_{1}, p_{2}, u\right)$, $h_{2}\left(p_{1}, p_{2}, u\right)$ so $E\left(p_{1}, p_{2}, u\right)=p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+p_{2} h_{2}\left(p_{1}, p_{2}, u\right)$.

## Intuition for the expenditure function

A student buys 100 packs of sandwiches a year. The sandwich price rises from $€ 1$ to $€ 1.50$. Could the student maintain the same level of utility with
€50 more?
$€ 60$ more?
$€ 40$ more?
€20 more?

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## $€ 50$ more? yes

$€ 60$ more?
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€20 more? ??

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## The logic of choice

Given a choice between grapes, cherries and apple you chose grapes.

Are you worse off if you are forced to chose between grapes, apricots \& blackberries?


## The logic of choice

Given a choice between grapes, cherries and apple you chose grapes.

Are you worse off if you are forced to chose between grapes, apricots \& blackberries?

No because you can still get grapes.

## Logic of choice

- If $€ 10$ is the cost of the cheapest way of doing something any other way of doing it costs $€ 10$ or more.
- If the range of things you can do changes, but you can still do what you did before you are no worse off.
- If prices change but you get just enough extra money to carry on doing the same thing you are no worse off.


## Homogeneity of the compensated demand and expenditure functions

## Mathematical definition of homogeneous functions

A function $f\left(z_{1}, z_{2}, z_{3} \ldots . z_{n}\right)$ is homogeneous of degree 0 if for all numbers $\mathrm{k}>0$
$f\left(k z_{1}, k z_{2}, k z_{3} \ldots . . k z_{n}\right)=k^{0} f\left(z_{1}, z_{2}, z_{3} \ldots \ldots z_{n}\right)=f\left(z_{1}, z_{2}, z_{3} \ldots \ldots z_{n}\right)$.
Multiplying $z_{1}, z_{2}, \ldots . . z_{n}$ by $k>0$ does not change the value of $f$.

A function $f\left(z_{1}, z_{2}, z_{3} \ldots . . z_{n}\right)$ is homogeneous of degree one if for all numbers $k>0$
$f\left(k z_{1}, k z_{2}, k z_{3} \ldots . . k z_{n}\right)=k^{1} f\left(z_{1}, z_{2}, z_{3} \ldots . . z_{n}\right)=k f\left(z_{1}, z_{2}, z_{3} \ldots . . z_{n}\right)$
Multiplying $z_{1}, z_{2} \ldots z_{n}$ by $k$ multiplies the value of $f$ by $k$.

## 3. Homogeneity of the compensated demand and expenditure functions

Compensated demand is homogeneous of degree 0 in prices.

If $\mathrm{k}>0 \quad \mathrm{~h}_{1}\left(\mathrm{kp}_{1}, \mathrm{kp}_{2}, \mathrm{u}\right)=\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$.
Expenditure function is homogeneous of degree 1 in prices.
If $\mathrm{k}>0 \quad \mathrm{E}\left(\mathrm{kp}_{1}, \mathrm{kp} \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{kE}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$.
The next slides explain why.


0

To get compensated demand fix utility and prices which fixes the indifference curve and gradient of budget line.
Get onto lowest possible budget line.

Compensated demand depends on the indifference curve and the slope $-p_{1} / p_{2}$ of the budget line.

Multiplying $p_{1}$ and $p_{2}$ by $k$ does not change the slope so does not change compensated demand so
$h_{1}\left(p_{1}, p_{2}, u\right)=h_{1}\left(k p_{1}, k p_{2}, u\right) \quad h_{2}\left(p_{1}, p_{2}, u\right)=h_{2}\left(k p_{1}, k p_{2}, u\right)$.
Compensated demand is homogeneous of degree 0 in prices.

From the definition of the expenditure function
$E\left(p_{1}, p_{2}, u\right)=p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+p_{2} h_{2}\left(p_{1}, p_{2}, u\right)$
\&
$E\left(k p_{1}, k p_{2}, u\right)=k p_{1} h_{1}\left(k p_{1}, k p_{2}, u\right)+k p_{2} h_{2}\left(k p_{1}, k p_{2}, u\right)$

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Since compensated demand is homogeneous of degree 0 in prices:

$$
\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{h}_{1}\left(k \mathrm{p}_{1}, k \mathrm{kp}_{2}, \mathrm{u}\right) \quad \mathrm{h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{h}_{2}\left(k \mathrm{pp}_{1}, \mathrm{kp}_{2}, \mathrm{u}\right) .
$$

From the definition of the expenditure function
$E\left(p_{1}, p_{2}, u\right)=p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+p_{2} h_{2}\left(p_{1}, p_{2}, u\right)$
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Since compensated demand is homogeneous of degree 0 in prices:

$$
\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{h}_{1}\left(k \mathrm{p}_{1}, k \mathrm{kp}_{2}, \mathrm{u}\right) \quad \mathrm{h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{h}_{2}\left(k \mathrm{pp}_{1}, k \mathrm{pp}_{2}, \mathrm{u}\right) .
$$

$$
E\left(k p_{1}, k p_{2}, u\right)=k p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+k p_{2} h_{2}\left(p_{1}, p_{2}, u\right)=
$$

$$
=k\left[p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+p_{2} h_{2}\left(p_{1}, p_{2}, u\right)\right]=k E\left(p_{1}, p_{2}, u\right)
$$

The expenditure function is homogeneous of degree 1 in prices.

Income and substitution effects and compensated and uncompensated demand

## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



## Income and substitution effects



# The logic of compensated demand and the expenditure function 

- By definition
compensated demand $\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right), \mathrm{h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$
is the cheapest way of getting utility $u$ at prices $p_{1}, p_{2}$
and costs $\mathrm{p}_{1} \mathrm{~h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)+\mathrm{p}_{2} \mathrm{~h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{E}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$
- Therefore any other way of getting utility u costs the same or more at prices $p_{1}, p_{2}$
- Thus if $u\left(x_{1}, x_{2}\right)=u$ then $E\left(p_{1}, p_{2}, u\right) \leq p_{1} x_{1}+p_{2} x_{2}$


# $\left(h_{1}, h_{2}\right)$ is by definition the cheapest way of getting utility $u$ at prices $\left(p_{1}, p_{2}\right)$. 

So at prices $\left(\mathrm{p}_{1}, \mathrm{p}_{2}\right)$
any other ( $\mathrm{x}_{1}, \mathrm{x}_{2}$ ) with utility u must lie on or above the budget line
$\mathrm{x}_{1}$ through $\left(\mathrm{h}_{1}, \mathrm{~h}_{2}\right)$ with slope - $p_{1} / p_{2}$

# $\left(h_{1}, h_{2}\right)$ is by definition the cheapest way of getting utility $u$ at prices $\left(p_{1}, p_{2}\right)$. 

So at prices $\left(p_{1}, p_{2}\right)$
any other ( $\mathrm{x}_{1}, \mathrm{x}_{2}$ ) with utility u cannot lie below the budget line
$\mathrm{x}_{1}$ through $\left(\mathrm{h}_{1}, \mathrm{~h}_{2}\right)$ with slope - $p_{1} / p_{2}$

Same information as in previous slide

Compensated demand curves cannot slope upwards: geometric proof

## 5. The slope of compensated demand

## curves

The substitution effect of an increase in the price of a good decreases or leaves unchanged the demand for the good.

The compensated demand curve can never slope upwards.


IMPORTANT RESULT

The next slides explain

## Geometric proof that the compensated demand curve cannot slope upwards

$h_{1 A}\left(p_{1 A}, p_{2}, u\right) \quad h_{2 A}\left(p_{1 A}, p_{2}, u\right)$ compensated demand with prices $p_{1 A}, p_{2}$ and utility $u$.

Therefore $u\left(h_{1 A}, h_{2 A}\right)=u$.
$h_{1 B}\left(p_{1 B}, p_{2}, u\right) \quad h_{2 B}\left(p_{1 B}, p_{2}, u\right)$ compensated demand with prices $p_{1 B}, p_{2}$ and utility $u$.

Therefore $u\left(h_{1 B}, h_{2 B}\right)=u$.
$h_{1 A}\left(p_{1 A}, p_{2}, u\right) \quad h_{2 A}\left(p_{1 A}, p_{2}, u\right)$ is the cheapest way of getting utility $u$ at prices $p_{1 A}, p_{2}$
$u\left(h_{1 B}, h_{2 B}\right)=u$ so $h_{1 B}, h_{2 B}$ is another way of getting utility $u$
therefore $h_{1 B}, h_{2 B}$ cannot cost less than $h_{1 A}, h_{2 A}$ at prices $p_{1 A}, p_{2}$
$h_{1 B}\left(p_{1 B}, p_{2}, u\right) \quad h_{2 B}\left(p_{1 B}, p_{2}, u\right)$ is the cheapest way of getting utility $u$ at prices $p_{1 B}, p_{2}$
$u\left(h_{1 A}, h_{2 A}\right)=u$ so $h_{1 A}, h_{2 A}$ is another way of getting utility $u$
therefore $h_{1 B}, h_{2 B}$ cannot cost more than $h_{1 A}, h_{2 A}$ at prices $\mathrm{p}_{1 \mathrm{~B}}, \mathrm{p}_{2}$

## Geometric proof that the compensated demand curve cannot slope upwards

$p_{1 A} h_{1 A}+p_{2} h_{2 A} \leq p_{1 A} h_{1 B}+p_{2} h_{2 B}$ because $h_{1 A}$ is cheapest at $p_{1 A}$
$p_{1 B} h_{1 B}+p_{2} h_{2 B} \leq p_{1 B} h_{1 A}+p_{2} h_{2 A}$ because $h_{1 B}$ is cheapest at $p_{1 B}$

$h_{1 B}, h_{2 B}$ cannot cost less $h_{1 A}, h_{2 A}$ at prices $p_{1 A}, p_{2}$
so $\left(h_{1 B}, h_{2 B}\right)$ cannot lie in the shaded area.

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Here $p_{1 B}>p_{1 A}$
$\left(\mathrm{h}_{1 \mathrm{~B}}, \mathrm{~h}_{2 \mathrm{~B}}\right)$ cannot lie in either shaded area
$\left(h_{1 B}, h_{2 B}\right)$ must lies in the white triangle, so $h_{1 B} \leq h_{1 A}$.
slope
$-\mathrm{p}_{1 \mathrm{~B}} / \mathrm{p}_{2}$
slope
$-p_{1 A} / p_{2}$

The compensated demand curve can never slope upwards.

Compensated demand curves cannot slope upwards: algebraic proof

## Algebraic proof that the compensated demand curve cannot slope upwards

$h_{1 A}\left(p_{1 A}, p_{2}, u\right) \quad h_{2 A}\left(p_{1 A}, p_{2}, u\right)$ is the cheapest way of getting utility $u$ at prices $p_{1 A}, p_{2}$
$u\left(h_{1 B}, h_{2 B}\right)=u$ so $h_{1 B}, h_{2 B}$ is another way of getting utility $u$

Therefore at prices $p_{1 A}, p_{2}$ quantities $h_{1 A}, h_{2 A}$ cannot cost more than $h_{1 B}, h_{2 B}$
so in algebra $p_{1 A} h_{1 A}+p_{2} h_{2 A} \leq p_{1 A} h_{1 B}+p_{2} h_{2 B}$
Remember the logic used here to derive the inequalities

## Algebraic proof that the compensated demand curve cannot slope upwards

$h_{1 B}\left(p_{1 B}, p_{2}, u\right) \quad h_{2 B}\left(p_{1 B}, p_{2}, u\right)$ is the cheapest way of getting utility $u$ at prices $p_{1 B}, p_{2}$
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Therefore at prices $p_{1 B}, p_{2}$ quantities $h_{1 B}, h_{2 B}$ cannot cost more than $h_{1 A}, h_{2 A}$.
so in algebra $p_{1 B} h_{1 B}+p_{2} h_{2 B} \leq p_{1 B} h_{1 A}+p_{2} h_{2 A}$
Remember the logic used here to derive the inequalities

## Algebraic proof that the compensated demand curve cannot slope upwards

Inequalities from the two previous slides

$$
\begin{aligned}
& p_{1 A} h_{1 A}+p_{2} h_{2 A} \leq p_{1 A} h_{1 B}+p_{2} h_{2 B} \\
& p_{1 B} h_{1 B}+p_{2} h_{2 B} \leq p_{1 B} h_{1 A}+p_{2} h_{2 A}
\end{aligned}
$$

Add the inequalities to get

$$
p_{1 A} h_{1 A}+p_{2} h_{2 A}+p_{1 B} h_{1 B}+p_{2} h_{2 B}
$$

$$
\leq p_{1 A} h_{1 B}+p_{2} h_{2 B}+p_{1 B} h_{1 A}+p_{2} h_{2 A}
$$

Remember the logic used here and derive the inequalities Everything that follows comes from simplifying this inequality.

Simplify the inequality from the previous slides

$$
p_{1 A} h_{1 A}+{ }_{2} h_{2 A}+p_{1 B} h_{1 B}+p_{2} h_{2 B}
$$

Subtract $p_{2} h_{2 A}$ and $p_{2} h_{2 B}$ from both sides to get
$p_{1 A} h_{1 A}+p_{1 B} h_{1 B} \leq p_{1 A} h_{1 B}+p_{1 B} h_{1 A}$

Remember the logic used here and derive the inequalities Everything that follows comes from simplifying this inequality.

From the previous slide
$p_{1 A} h_{1 A}+p_{1 B} h_{1 B} \leq p_{1 A} h_{1 B}+p_{1 B} h_{1 A}$
Rearrange to get $0 \leq\left(p_{1 B}-p_{1 A}\right)\left(h_{1 A}-h_{1 B}\right)$
so if the price rises from $p_{1 A}$ to $p_{1 B}$ so $p_{1 B}-p_{1 A}>0$
either $h_{1 A}-h_{1 B}=0$ so compensated demand does not change
or $h_{1 A}-h_{1 B}>0$ so compensated demand falls.

Compensated demand and the expenditure function with CobbDouglas utility
6. Compensated demand and the expenditure function with Cobb-Douglas utility

## Step 1: What problem are you solving?

The problem is minimising expenditure $\mathrm{p}_{1} \mathrm{x}_{1}+\mathrm{p}_{2} \mathrm{x}_{2}$ subject to non-negativity constraints $\mathrm{x}_{1} \geq 0 \quad \mathrm{x}_{2} \geq 0$ and the utility constraint $u\left(x_{1}, x_{2}\right)=x_{1}{ }^{2 / 5} x_{2}{ }^{3 / 5} \geq u$.

Step 2: What is the solution a function of?
Compensated demand is a function of prices \&utility so is $\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right) \quad \mathrm{h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$

## Finding compensated demand with CobbDouglas utility

Step 3: Check for nonsatiation and convexity
We have already done this, both are satisfied.

Why does this matter?
See the next slide.

## Essential logic

## With nonsatiation and convexity


if there is a tangency point such as A
where MRS = $p_{1} / p_{2}$
and utility is $u$
this is compensated demand
because any cheaper point such as B gives lower utility.

## Finding compensated demand with CobbDouglas utility

Step 4: Use the tangency and utility conditions

Tangency requires that
MRS =
$\frac{p_{1}}{p_{2}}$
here we have already found
$\mathrm{MRS}=\quad \frac{\frac{\partial u}{\partial x_{1}}}{\frac{\partial u}{\partial x_{2}}}=\frac{\frac{2}{5} x_{1}^{-3 / 5} x_{2}^{3 / 5}}{\frac{3}{5} x_{1}^{2 / 5} x_{2}^{-2 / 5}}=\frac{2 x_{2}}{3 x_{1}}$

## Finding compensated demand with CobbDouglas utility

## Step 4: Use the tangency and utility conditions

tangency condition

$$
\frac{2 x_{2}}{3 x_{1}}=\frac{p_{1}}{p_{2}}
$$

utility condition

$$
x_{1}^{2 / 5} x_{2}^{3 / 5}=u
$$

because if $x_{1}^{2 / 5} x_{2}^{3 / 5}>u$ there is a cheaper way of getting utility u or more.

## Finding compensated demand with CobbDouglas utility

Step 5: Draw a diagram based on the tangency and utility conditions


## Finding compensated demand with CobbDouglas utility

Step 6: Remind yourself what you are finding and what it depends on.
You are finding compensated demand $h_{1}$ and $h_{2}$ which are functions of $p_{1}, p_{2}$ and $u$.

Step 7: Write down the equations to be solved.

The equations are

$$
\begin{aligned}
x_{1}^{2 / 5} x_{2} 3 / 5 & =u \quad \text { and } \\
\frac{2 x_{2}}{3 x_{1}} & =\frac{p_{1}}{p_{2}}
\end{aligned}
$$

## Finding compensated demand with CobbDouglas utility

Step 8 solve the equations and write down the solution as a function.
This gives $x_{1}=\left(\frac{2 p_{2}}{3 p_{1}}\right)^{3 / 5} u \quad x_{2}=\left(\frac{3 p_{1}}{2 p_{2}}\right)^{2 / 5} u$ usingnotation $\quad h_{1}\left(p_{1}, p_{2}, u\right) \quad h_{2}\left(p_{1}, p_{2}, u\right)$ for compensated demand
$h_{1}\left(p_{1}, p_{2}, u\right)=\left(\frac{2 p_{2}}{3 p_{1}}\right)^{3 / 5} u \quad h_{2}\left(p_{1}, p_{2}, u\right)=\left(\frac{3 p_{1}}{2 p_{2}}\right)^{2 / 5} u$.
Note $h_{1}\left(p_{1}, p_{2}, u\right)>0$,
$h_{2}\left(p_{1}, p_{2}, u\right)>0$.

## Uncompensated and compensated demand with Cobb-Douglas utility

Uncompensated demand
$x_{1}\left(p_{1}, p_{2}, m\right)=\frac{2}{5} \frac{m}{p_{1}} \quad x_{2}\left(p_{1}, p_{2}, m\right)=\frac{3}{5} \frac{m}{p_{2}}$
Compensated demand
$h_{1}\left(p_{1}, p_{2}, u\right)=\left(\frac{2 p_{2}}{3 p_{1}}\right)^{3 / 5} u \quad h_{2}\left(p_{1}, p_{2}, u\right)=\left(\frac{3 p_{1}}{2 p_{2}}\right)^{2 / 5} u$.

## The expenditure function with Cobb-Douglas utility

Compensated demand is
$h_{1}\left(p_{1}, p_{2}, u\right)=\left(\frac{2 p_{2}}{3 p_{1}}\right)^{3 / 5} u \quad h_{2}\left(p_{1}, p_{2}, u\right)=\left(\frac{3 p_{1}}{2 p_{2}}\right)^{2 / 5} u$
so the expenditure function is

$$
E\left(p_{1}, p_{2}, u\right)=p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+p_{2} h_{2}\left(p_{1}, p_{2}, u\right)
$$

$=p_{1}\left(\frac{2 p_{2}}{3 p_{1}}\right)^{3 / 5} u+p_{2}\left(\frac{3 p_{1}}{2 p_{2}}\right)^{2 / 5} u$
$=p_{1}^{2 / 5} p_{2}^{3 / 5}\left(\left(\frac{2}{3}\right)^{3 / 5}+\left(\frac{3}{2}\right)^{2 / 5}\right) u$

## Check

Compensated demand is function of prices and utility.
It is homogeneous of degree 0 in prices.

The expenditure function is a function of prices and utility.
It is homogeneous of degree 1 in prices.

## Income and substitution effects with Cobb-Douglas utility

## Income and substitution effects with CobbDouglas utility



## Income and substitution effects with CobbDouglas utility



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## Uncompensated and compensated demand with Cobb-Douglas utility

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$x_{1}\left(p_{1}, p_{2}, m\right)=\frac{2}{5} \frac{m}{p_{1}} \quad x_{2}\left(p_{1}, p_{2}, m\right)=\frac{3}{5} \frac{m}{p_{2}}$
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$h_{1}\left(p_{1}, p_{2}, u\right)=\left(\frac{2 p_{2}}{3 p_{1}}\right)^{3 / 5} u \quad h_{2}\left(p_{1}, p_{2}, u\right)=\left(\frac{3 p_{1}}{2 p_{2}}\right)^{2 / 5} u$.

## Uncompensated and compensated demand with Cobb-Douglas utility

income effect: change in m on uncompensated demand

Uncompensated demand
$x_{1}\left(p_{1}, p_{2}, m\right)=\frac{2}{5} \frac{m}{p_{1}} \quad x_{2}\left(p_{1}, p_{2}, m\right)=\frac{3}{5} \frac{m}{p_{2}}$
Compensated demand
$h_{1}\left(p_{1}, p_{2}, u\right)=\left(\frac{2 p_{2}}{3 p_{1}}\right)^{3 / 5} u \quad h_{2}\left(p_{1}, p_{2}, u\right)=\left(\frac{3 p_{1}}{2 p_{2}}\right)^{2 / 5} u$.

## Uncompensated and compensated demand with Cobb-Douglas utility

income effect: change in m on uncompensated demand

Uncompensated demand
$x_{1}\left(p_{1}, p_{2}, m\right)=\frac{2}{5} \frac{m}{p_{1}} \quad x_{2}\left(p_{1}, p_{2}, m\right)=\frac{3}{5} \frac{m}{p_{2}}$
Compensated demand
$h_{1}\left(p_{1}, p_{2}, u\right)=\left(\frac{2 p_{2}}{p_{1}}\right)^{3 / 5} u \quad h_{2}\left(p_{1}, p_{2}, u\right)=\left(\frac{3\left(p_{1}\right)}{2 p_{2}}\right)^{2 / 5} u$.
substitution effect: change on $\mathrm{p}_{1}$ on compensated demand

## Income and substitution effects with Cobb-Douglas utility

income effect $\frac{\partial x_{1}}{\partial m}=$

substitution effect $\frac{\partial h_{1}}{\partial p_{1}}=$


## Income and substitution effects with Cobb-Douglas utility

income effect $\frac{\partial x_{1}}{\partial m}=\frac{2}{5} \frac{1}{p_{1}}>0$
substitution effect $\frac{\partial h_{1}}{\partial p_{1}}=-\left(\frac{3}{5}\right)\left(\frac{2 p_{2}}{3}\right)^{3 / 5} p_{1}^{-8 / 5} u<0$

Compensated demand and the expenditure function with perfect complements utility

## 7. Compensated demand and the expenditure function with perfect complements utility

$u\left(x_{1}, x_{2}\right)=\min \left(1 / 2 x_{1}, x_{2}\right)$

$x_{1}$ bicycle wheels, $x_{2}$ bicycle frames
if $\mathrm{X}_{2}<1 / 2 \mathrm{X}_{1}$ increasing $\mathrm{x}_{1}$ does not change utility
if $x_{2}>1 / 2 x_{1}$ increasing $x_{1}$ increases utility.

Finding compensated demand with perfect complements utility

$$
u\left(x_{1}, x_{2}\right)=\min \left(1 / 2 x_{1}, x_{2}\right)=u
$$

$\min \left(1 / 2 x_{1}, x_{2}\right)=u$


Finding compensated demand with perfect complements utility

$$
u\left(x_{1}, x_{2}\right)=\min \left(1 / 2 x_{1}, x_{2}\right)=u
$$

$\min \left(1 / 2 x_{1}, x_{2}\right)=u$


Expenditure minimization implies that $\left(x_{1}, x_{2}\right)$ lies at the corner of the indifference curves so
$x_{2}=1 / 2 x_{1}$
and gives utility u so
$1 / 2 x_{1}=x_{2}=u$.
$\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=2 \mathrm{u}, \mathrm{h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{u}$.

## Uncompensated and compensated demand with perfect complements utility

uncompensated demand
$x_{1}\left(p_{1}, p_{2}, m\right)=\frac{2 m}{2 p_{1}+p_{2}}, \quad x_{2}\left(p_{1}, p_{2}, m\right)=\frac{m}{2 p_{1}+p_{2}}$
$h_{1}\left(p_{1}, p_{2}, u\right)=2 \mathrm{u} \quad h_{2}\left(p_{1}, p_{2}, u\right)=u$
compensated demand

## Income and substitution effects with perfect complements utility

# Income and substitution effects with perfect complements utility 

## income effect

substitution effect



# Income and substitution effects with perfect complements utility 

income effect $\frac{\partial x_{1}}{\partial m}=\frac{2}{2 p_{1}+p_{2}}>0$
substitution effect $\frac{\partial h_{1}}{\partial p_{1}}=0$

## Income and substitution effects with perfect complements utility

$$
\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=2 \mathrm{u}, \mathrm{~h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{u} .
$$



## Income and substitution effects with perfect complements utility



Income and substitution effects with perfect complements utility


Income and substitution effects with perfect complements utility


Income and substitution effects with perfect complements utility


Income and substitution effects with perfect complements utility


Income and substitution effects with perfect complements utility


## The expenditure function with perfect complements utility

Compensated demand is
$h_{1}\left(p_{1}, p_{2}, u\right)=2 u \quad h_{2}\left(p_{1}, p_{2}, u\right)=u$
so the expenditure function is

$$
\begin{aligned}
& E\left(p_{1}, p_{2}, u\right)=p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+p_{2} h_{2}\left(p_{1}, p_{2}, u\right) \\
& =p_{1} 2 u+p_{2} u=\left(2 p_{1}+p_{2}\right) u
\end{aligned}
$$

# Properties of the expenditure function 

## 8. Properties of the Expenditure Function

1. Increasing in utility
2. The expenditure function increases or does not change when a price increases.
3. Homogeneous of degree 1 in prices.
4. Shephard's lemma $\frac{\partial E\left(p_{1}, p_{2}, u\right)}{\partial p_{1}}=h_{1}\left(p_{1}, p_{2}, u\right)$

## Definition of the expenditure function

The expenditure function $E\left(p_{1}, p_{2}, u\right)$ is the minimum amount of money you have to spend to get utility $u$ with prices $p_{1}$ and $p_{2}$. It is a function of $p_{1}, p_{2}$ and $u$.

The amount of goods which minimizes the cost of getting utility $u$ is compensated demand $h_{1}\left(p_{1}, p_{2}, u\right), h_{2}\left(p_{1}, p_{2}, u\right)$
so $E\left(p_{1}, p_{2}, u\right)=p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+p_{2} h_{2}\left(p_{1}, p_{2}, u\right)$.

1. The expenditure function is increasing in utility


Utility increases from $u_{1}$ to $u_{2}$.
The expenditure function increases.
2. The expenditure function increases or does not change when prices increase


The price of good 1 increases from $p_{1 A}$ to $p_{1 B}$, compensated demand moves from $A$ to $B$.

The expenditure function increases.

The expenditure function does not change if demand for good 1 is 0 at $A$.

## 3: The expenditure function is homogeneous of degree 1 in prices.

Compensated demand is homogeneous of degree 0 in prices.

If $k>0 \quad h_{1}\left(k_{1}, k p_{2}, u\right)=h_{1}\left(p_{1}, p_{2}, u\right)$.
Expenditure function is homogeneous of degree 1 in prices.
If $\mathrm{k}>0 \quad \mathrm{E}\left(\mathrm{kp}_{1}, \mathrm{kp} \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{kE}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$.

Already explained.

# Properties of the expenditure function 4 Shephard's Lemma <br> $\partial \mathrm{E}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$ <br> $$
=h_{1}\left(p_{1}, p_{2}, u\right)
$$ 

## 4. Shephard's Lemma

$$
\frac{\partial \mathrm{E}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)}{\partial \mathrm{p}_{1}}=\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)
$$

derivative of expenditure $=$ compensated function w.r.t. $p_{1}$ demand for good 1

u constant
$\mathrm{p}_{2}$ constant
$\mathrm{p}_{1}$ varies

- By definition
compensated demand $\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right), \mathrm{h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$
is the cheapest way of getting utility $u$ at prices $p_{1}, p_{2}$
and costs $\mathrm{p}_{1} \mathrm{~h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)+\mathrm{p}_{2} \mathrm{~h}_{2}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)=\mathrm{E}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$
- Therefore any other way of getting utility u costs the same or more at prices $p_{1}, p_{2}$
- Thus if $u\left(x_{1}, x_{2}\right)=u$ then $E\left(p_{1}, p_{2}, u\right) \leq p_{1} x_{1}+p_{2} x_{2}$
$h_{1 A}\left(p_{1 A}, p_{2}, u\right) \quad h_{2 A}\left(p_{1 A}, p_{2}, u\right)$ compensated demand with prices $p_{1 A}, p_{2}$ and utility $u$.

Therefore $u\left(h_{1 A}, h_{2 A}\right)=u$ and
$p_{1 A} h_{1 A}+p_{2} h_{2 A}=E\left(p_{1 A}, p_{2}, u\right)$
$h_{1}\left(p_{1}, p_{2}, u\right) \quad h_{2}\left(p_{1}, p_{2}, u\right)$ compensated demand with prices $p_{1}, p_{2}$ and utility $u$.

Therefore $u\left(h_{1}, \mathrm{~h}_{2}\right)=\mathrm{u}$
and $p_{1} h_{1}\left(p_{1}, p_{2}, u\right)+p_{2} h_{2}\left(p_{1}, p_{2}, u\right)=E\left(p_{1}, p_{2}, u\right)$
$E\left(p_{1}, p_{2}, u\right)$ is the cheapest way of getting utility $u$ at prices $\left(p_{1}, p_{2}\right)$
$\left(h_{1 A}, h_{2 A}\right)$ is another way of getting utility $u$, and at prices $\left(p_{1}, p_{2}\right)$ costs $p_{1} h_{1 A}+p_{2} h_{2 A}$ so
$E\left(p_{1}, p_{2}, u\right) \leq p_{1} h_{1 A}+p_{2} h_{2 A}$ for all $p_{1}$
$E\left(p_{1 A}, p_{2}, u\right)=p_{1 A} h_{1 A}+p_{2} h_{2 A}$

## New diagram

In this diagram $\left(h_{1 A}, h_{2 A}\right) u$ and $p_{2}$ do not vary, $p_{1}$ varies.

Cost of buying ( $h_{1 A}, h_{2 A}$ ) at prices $\left(p_{1}, p_{2}\right)$ is $p_{1} h_{1 A}+p_{2} h_{2 A}$

$E\left(p_{1}, p_{2}, u\right) \leq p_{1} h_{1 A}+p_{2} h_{2 A}$ for all $p_{1}$
$E\left(p_{1 A}, p_{2}, u\right)=p_{1 A} h_{1 A}+p_{2} h_{2 A}$.
The graph of $\mathrm{E}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$ cannot be anywhere inside the shaded area.


The graph of $E\left(p_{1}, p_{2}, u\right)$ meets the line with slope $h_{1 A}$ at $A$ and is never above the line.
so the line is tangent to the graph of $\mathrm{E}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)$ at A
so the derivative of $E\left(p_{1}, p_{2}, u\right)$ with respect to $p_{1}$ at $A$ is $h_{1 A}$ (compensated demand for good 1)


## Shepard's Lemma

The derivative of the expenditure function
$E\left(p_{1}, p_{2}, u\right)$ with respect to $p_{1}$
is compensated demand for good 1

$$
\frac{\partial \mathrm{E}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)}{\partial \mathrm{p}_{1}}=\mathrm{h}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{u}\right)
$$

Try with previous examples

## The Slutsky equation

## 9. The Slutsky equation

$$
\text { When } \quad m=E\left(p_{1}, p_{2}, u\right)
$$



$$
=\frac{\partial h_{1}\left(p_{1}, p_{2}, u\right)}{\partial p_{1}}-\frac{\partial x_{1}\left(p_{1}, p_{2}, m\right)}{\partial m} x_{1}\left(p_{1}, p_{2}, m\right)
$$

substitution effect of change in price on demand, at constant utility
income effect of
change in
income on
demand

## Why bother with the Slutsky Equation?

It is the only way of knowing how big income and substitution effects are.

Combined with elasticity estimates it tells us that income effects are too small to bother with except for goods that are a large proportion of the budget.
$h_{1}\left(p_{1}, p_{2}, u\right) \quad$ compensated demand for good 1.
$x_{1}\left(p_{1}, p_{2}, m\right)$ uncompensated demand for good 1.
$A$ to $B$, change in compensated demand. This is the substitution effect.
$B$ to $C$ income effect. This is the shift in uncompensated demand when income m changes.

## Intuition for the Slutsky Equation

When $p_{1}$ rises to $p_{1}+\Delta p_{1}$ this is as if income falls by $x_{1} \Delta p_{1}$ because this is how much more it costs to buy $x_{1}$, so the effective change in income is $\Delta m=-x_{1} \Delta p_{1}$.

The change in $\mathrm{x}_{1}$ through the income effect is approximately

$$
\frac{\partial x_{1}}{\partial m} \Delta m=-\frac{\partial x_{1}}{\partial m} x_{1} \Delta p_{1}
$$

The change in $\mathrm{x}_{1}$ through the substitution effect is approximately
Total change

$$
\begin{aligned}
& \Delta x_{1} \approx \frac{\partial h_{1}}{\partial p_{1}} \Delta p_{1} \quad-\frac{\partial x}{\partial r} \\
& \frac{\partial x_{1}}{\partial p_{1}}=\frac{\partial h_{1}}{\partial p_{1}} \quad-\frac{\partial x_{1}}{\partial m} x_{1}
\end{aligned}
$$

so

When $m=E\left(p_{1}, p_{2}, u\right)$
$\frac{\partial x_{1}\left(p_{1}, p_{2}, m\right)}{\partial p_{1}}=\frac{\partial h_{1}\left(x_{1}, x_{2}, u\right)}{\partial p_{1}}-\frac{\partial x_{1}\left(p_{1}, p_{2}, m\right)}{\partial m} x_{1}\left(p_{1}, p_{2}, m\right)$
leave out the arguments to make this easier to write down
$\frac{\partial x_{1}}{\partial p_{1}}=\frac{\partial h_{1}}{\partial p_{1}}-\frac{\partial x_{1}}{\partial m} x_{1}$
multiply by $p_{1} / x_{1}=p_{1} / h_{1}$ because $x_{1}=h_{1}$
$\frac{p_{1}}{x_{1}} \frac{\partial x_{1}}{\partial p_{1}}$
$=\frac{p_{1}}{h_{1}} \frac{\partial h_{1}}{\partial p_{1}}$
$-\frac{m}{x_{1}} \frac{\partial x_{1}}{\partial m} \frac{p_{1} x_{1}}{m}$
Slutsky equation in elasticities

$$
\frac{p_{1}}{x_{1}} \frac{\partial x_{1}}{\partial p_{1}}=\frac{p_{1}}{h_{1}} \frac{\partial h_{1}}{\partial p_{1}}-\frac{m}{x_{1}} \frac{\partial x_{1}}{\partial m} \frac{p_{1} x_{1}}{m}
$$

## But what are these elasticities?



$\frac{p_{1}}{x_{1}} \frac{\partial x_{1}}{\partial p_{1}}=\frac{p_{1}}{h_{1}} \frac{\partial h_{1}}{\partial p_{1}}$




Perloff notation

## Income effects and the Slutsky Equation



Sign?


Sign for a normal good

Sign for an inferior good

## Income effects and the Slutsky Equation

$$
\frac{p_{1}}{x_{1}} \frac{\partial x_{1}}{\partial p_{1}}=\frac{p_{1}}{h_{1}} \frac{\partial h_{1}}{\partial p_{1}}-\left(\frac{m}{x_{1}} \frac{\partial x_{1}}{\partial m} \frac{p_{1} x_{1}}{m}\right.
$$

Sign?
negative
Sign for a normal good

Sign for an inferior good

## Income effects and the Slutsky Equation

$\frac{p_{1}}{x_{1}} \frac{\partial x_{1}}{\partial p_{1}}=\frac{p_{1}}{h_{1}} \frac{\partial h_{1}}{\partial p_{1}}$
Sign?
negative


Sign for a normal good positive

Sign for an inferior good

## Income effects and the Slutsky Equation

$\frac{p_{1}}{x_{1}} \frac{\partial x_{1}}{\partial p_{1}}=\quad \frac{p_{1}}{h_{1}} \frac{\partial h_{1}}{\partial p}$
Sign?
negative


Sign for a normal good positive

Sign for an inferior good negative.

## Income effects and the Slutsky Equation

$$
\frac{p_{1}}{x_{1}} \frac{\partial x_{1}}{\partial p_{1}}=\frac{p_{1}}{h_{1}} \frac{\partial h_{1}}{\partial p_{1}}-(\frac{m}{x_{1}} \frac{\partial x_{1}}{\partial m} \underbrace{p_{1} x_{1}}_{\text {income elasticity }}
$$

Income effects are small for goods with a small budget share, even if the income elasticity of demand is quite large.

Quantitatively income effects often don't matter.

## Elasticity of demand curves



Which demand curve is more elastic A or B ?
If good 1 is a normal good which is more elastic, compensated or uncompensated demand?

If good 1 is an inferior good which is more elastic, compensated or uncompensated demand?

## Elasticity of demand curves



Which demand curve is more elastic A or $\underline{B}$ ?
If good 1 is a normal good which is more elastic, compensated or uncompensated demand?

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## Elasticity of demand curves



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## Elasticity of demand curves



Which demand curve is more elastic A or $\underline{B}$ ?
If good 1 is a normal good which is more elastic, compensated or uncompensated demand?

If good 1 is an inferior good which is more elastic,
compensated or uncompensated demand?

## Expenditure minimization \& compensated demand. What have we achieved?

- Income and substitution effects. Will be important for
- labour supply, saving \& borrowing.
- Downward sloping compensated demand
- Result from the Slutsky equation, income effect is small if budget share is small.
- Foundation for understanding welfare measures
- consumer surplus, price indices, compensating and equivalent variation.

