Economics Lecture 4

2016-17

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Course Outline

- 1 Consumer theory and its applications
 - 1.1 Preferences and utility
 - 1.2 Utility maximization and uncompensated demand
 - 1.3 Expenditure minimization and compensated demand
 - 1.4 Price changes and welfare
 - 1.5 Labour supply, taxes and benefits
 - 1.6 Saving and borrowing

2 Firms, costs and profit maximization

- 2.1 Firms and costs
- 2.2 Profit maximization and costs for a price taking firm
- 3. Industrial organization
 - 3.1 Perfect competition and monopoly
 - 3.2 Oligopoly and games

1.3 Expenditure minimization and compensated demand

1.3 Expenditure minimization and compensated demand

- 1. Definitions of compensated & uncompensated demand
- 2. Definition of the expenditure function
- 3. Homogeneity of the compensated demand and expenditure functions
- 4. Income & substitution effects
- 5. The slope of compensated demand curves
- 6. Compensated demand & the expenditure function with Cobb-Douglas utility

Expenditure minimization and compensated demand

- 7. Compensated demand & the expenditure function with perfect complements and perfect substitutes utility
- 8. Properties of the expenditure function
- 9. The Slutsky equation

Expenditure minimization and compensated demand

Why bother?

 Income and substitution effects, essential for understanding the effects of changes in wages and taxes on labour supply and interest rates on savings.

2. Expenditure function, essential for consumer surplus and welfare economics.

Definitions of compensated and uncompensated demand

1. Definitions of compensated and uncompensated demand

What we have been calling demand up to now is

uncompensated (Marshallian) demand which <u>maximizes</u> <u>utility</u> u given prices p_1 and p_2 and income m, so is a function of p_1 , p_2 , m,

notation $x_1(p_1, p_2, m)$, $x_2(p_1, p_2, m)$.

Compensated (Hicksian) demand <u>minimizes</u> the cost of obtaining utility u at prices p_1 and p_2 and is a function of utility u, p_1 , p_2 , notation $h_1(p_1, p_2, u)$, $h_2(p_1, p_2, u)$.



Get onto highest possible indifference curve.

To get <u>compensated</u> demand fix <u>utility</u> and <u>prices</u> which fixes the indifference curve and gradient of budget line.

Get onto lowest possible budget

line.



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Get onto lowest possible budget

Definition of the expenditure function and intuition

2. Definition of the expenditure function

The **expenditure function** E (p_1 , p_2 ,u) is the minimum amount of money you have to spend to get utility u with prices p_1 and p_2 . It is a function of p_1 , p_2 and u.

The amount of goods which minimizes the cost of getting utility u is compensated demand h_1 (p_1 , p_2 ,u), h_2 (p_1 , p_2 ,u)

so $E(p_1, p_2, u) = p_1h_1(p_1, p_2, u) + p_2h_2(p_1, p_2, u)$.

A student buys 100 packs of sandwiches a year. The sandwich price rises from €1 to €1.50. Could the student maintain the same level of utility with

€50 more?

€60 more?

€40 more?

€20 more?





A student buys 100 packs of sandwiches a year. The sandwich price rises from €1 to €1.50. Could the student maintain the same level of utility with

€50 more? yes

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€40 more? ?

€20 more? ??





The logic of choice

Given a choice between grapes, cherries and apple you chose grapes.

Are you worse off if you are forced to chose between grapes, apricots & blackberries?





The logic of choice

Given a choice between grapes, cherries and apple you chose grapes.

Are you worse off if you are forced to chose between grapes, apricots & blackberries?

No because you can still get grapes.

Logic of choice

- If €10 is the cost of the cheapest way of doing something any other way of doing it costs €10 or more.
- If the range of things you can do changes, but you can still do what you did before you are no worse off.
- If prices change but you get just enough extra money to carry on doing the same thing you are no worse off.

Homogeneity of the compensated demand and expenditure functions

Mathematical definition of homogeneous functions

A function $f(z_1, z_2, z_3, ..., z_n)$ is <u>homogeneous of degree 0</u> if for all numbers k > 0

 $f(kz_1, kz_2, kz_3, \dots, kz_n) = k^0 f(z_1, z_2, z_3, \dots, z_n) = f(z_1, z_2, z_3, \dots, z_n).$

Multiplying z_1, z_2, \dots, z_n by k > 0 does not change the value of f.

A function $f(z_1, z_2, z_3, ..., z_n)$ is <u>homogeneous of degree one</u> if for all numbers k > 0

 $f(kz_1, kz_2, kz_3, \dots, kz_n) = k^1 f(z_1, z_2, z_3, \dots, z_n) = kf(z_1, z_2, z_3, \dots, z_n)$

Multiplying $z_1, z_2 \dots z_n$ by k multiplies the value of f by k.

3. Homogeneity of the compensated demand and expenditure functions

Compensated demand is homogeneous of degree 0 in prices.

If
$$k > 0$$
 $h_1(kp_1, kp_2, u) = h_1(p_1, p_2, u)$.

Expenditure function is homogeneous of degree 1 in prices.

If
$$k > 0$$
 $E(kp_1, kp_2, u) = k E(p_1, p_2, u)$.

The next slides explain why.



To get <u>compensated</u> demand fix <u>utility</u> and <u>prices</u> which fixes the indifference curve and gradient of budget line.

Get onto lowest possible budget line.

Compensated demand depends on the indifference curve and the slope $-p_1/p_2$ of the budget line.

Multiplying p_1 and p_2 by k does not change the slope so does not change compensated demand so

 $h_1(p_1,p_2,u) = h_1(kp_1,kp_2,u)$ $h_2(p_1,p_2,u) = h_2(kp_1,kp_2,u).$

Compensated demand is homogeneous of degree 0 in prices.

From the definition of the expenditure function

 $E(kp_1, kp_2, u) = kp_1h_1(kp_1, kp_2, u) + kp_2h_2(kp_1, kp_2, u)$

From the definition of the expenditure function

$$E(p_1, p_2, u) = p_1 h_1(p_1, p_2, u) + p_2 h_2(p_1, p_2, u)$$

&

E(kp₁, kp₂,u)= kp₁h₁(kp₁, kp₂,u) + kp₂ h₂(kp₁, kp₂,u) Since compensated demand is homogeneous of degree 0 in prices:

 $h_1(p_1,p_2,u) = h_1(kp_1,kp_2,u)$ $h_2(p_1,p_2,u) = h_2(kp_1,kp_2,u).$
From the definition of the expenditure function

$$E(p_1, p_2, u) = p_1 h_1(p_1, p_2, u) + p_2 h_2(p_1, p_2, u)$$

&

E(kp₁, kp₂,u)= kp₁h₁(kp₁, kp₂,u) + kp₂ h₂(kp₁, kp₂,u) Since compensated demand is homogeneous of degree 0 in prices:

$$\begin{split} h_1(p_1,p_2,u) &= h_1(kp_1,kp_2,u) & h_2(p_1,p_2,u) = h_2(kp_1,kp_2,u). \\ E(kp_1, kp_2,u) &= kp_1h_1(p_1, p_2,u) + kp_2 h_2(p_1, p_2,u) = \\ &= k \left[p_1h_1(p_1, p_2,u) + p_2 h_2(p_1, p_2,u) \right] = k E(p_1, p_2,u) \end{split}$$

The expenditure function is homogeneous of degree 1 in prices.

Income and substitution effects and compensated and uncompensated demand



























Х₁



X₂

The logic of compensated demand and the expenditure function • By definition

compensated demand $h_1(p_1,p_2,u)$, $h_2(p_1,p_2,u)$

is the cheapest way of getting utility u at prices $p_1,\,p_2$

and costs $p_1 h_1(p_1,p_2,u) + p_2 h_2(p_1,p_2,u) = E(p_1,p_2,u)$

- Therefore any other way of getting utility u costs the same or more at prices p₁, p₂
- Thus if $u(x_1, x_2) = u$ then $E(p_1, p_2, u) \le p_1 x_1 + p_2 x_2$



 (h_1,h_2) is by definition the cheapest way of getting utility u at prices (p_1, p_2) .

So at prices (p_1, p_2) any other (x_1, x_2) with utility u <u>must lie</u> on or <u>above</u> the budget line through (h_1, h_2) with slope - p_1/p_2



 (h_1,h_2) is by definition the cheapest way of getting utility u at prices (p_1, p_2) .

So at prices (p_1, p_2) any other (x_1, x_2) with utility u <u>cannot lie</u> <u>below</u> the budget line through (h_1, h_2) with slope $-p_1/p_2$

Same information as in previous slide

Compensated demand curves cannot slope upwards: geometric proof

5. The slope of compensated demand

curves

The <u>substitution effect</u> of an <u>increase</u> in the price of a good <u>decreases or leaves unchanged</u> the demand for the good.

The compensated demand curve can never slope upwards.



IMPORTANT RESULT

The next slides explain

Geometric proof that the compensated demand curve cannot slope upwards

 $h_{1A}(p_{1A},p_2,u)$ $h_{2A}(p_{1A},p_2,u)$ compensated demand with

prices p_{1A} , p_2 and utility u.

Therefore $u(h_{1A}, h_{2A}) = u$.

 $h_{1B}(p_{1B}, p_2, u)$ $h_{2B}(p_{1B}, p_2, u)$ compensated demand with

prices p_{1B} , p_2 and utility u.

Therefore $u(h_{1B}, h_{2B}) = u$.

 $h_{1A}(p_{1A},p_2,u) \quad h_{2A}(p_{1A},p_2,u)$ is the <u>cheapest</u> way of getting utility u at prices p_{1A},p_2 $u(h_{1B},h_{2B}) = u \text{ so } h_{1B},h_{2B}$ is <u>another way</u> of getting utility u

therefore h_{1B} , h_{2B} cannot cost less than h_{1A} , h_{2A} at prices p_{1A} , p_2

 $h_{1B}(p_{1B},p_2,u)$ $h_{2B}(p_{1B},p_2,u)$ is the <u>cheapest</u> way of getting utility u at prices p_{1B},p_2 $u(h_{1A},h_{2A}) = u$ so h_{1A},h_{2A} is <u>another way</u> of getting utility u

therefore $h_{1\text{B}},h_{2\text{B}}$ cannot cost more than $h_{1\text{A}},h_{2\text{A}}$ at prices $p_{1\text{B}},p_{2}$

Geometric proof that the compensated demand curve cannot slope upwards

 $p_{1A}h_{1A} + p_2h_{2A} \le p_{1A}h_{1B} + p_2h_{2B}$ because h_{1A} is cheapest at p_{1A} $p_{1B}h_{1B} + p_2h_{2B} \le p_{1B}h_{1A} + p_2h_{2A}$ because h_{1B} is cheapest at p_{1B}
























(h_{1B} , h_{2B}) <u>**must**</u> lies in the white triangle, so $h_{1B} \leq h_{1A}$.



The compensated demand curve can never slope upwards.

Here $p_{1B} > p_{1A}$

 $(h_{1B}, h_{2B}) \frac{cannot}{must}$ lie in either shaded area $(h_{1B}, h_{2B}) \frac{must}{must}$ lies in the white triangle, so $h_{1B} \leq h_{1A}$.

Compensated demand curves cannot slope upwards: algebraic proof

Algebraic proof that the compensated demand curve cannot slope upwards

 $h_{1A}(p_{1A},p_2,u)$ $h_{2A}(p_{1A},p_2,u)$ is the <u>cheapest</u> way of getting

utility u at prices p_{1A}, p_2

 $u(h_{1B},h_{2B}) = u \text{ so } h_{1B},h_{2B}$ is <u>another way</u> of getting utility u

Therefore at prices p_{1A} , p_2 quantities h_{1A} , h_{2A} cannot cost more than h_{1B} , h_{2B}

so in algebra $p_{1A}h_{1A} + p_2h_{2A} \le p_{1A}h_{1B} + p_2h_{2B}$

Remember the logic used here to derive the inequalities

Algebraic proof that the compensated demand curve cannot slope upwards

 $h_{1B}(p_{1B},p_2,u)$ $h_{2B}(p_{1B},p_2,u)$ is the <u>cheapest</u> way of getting

utility u at prices p_{1B}, p_2

 $u(h_{1A},h_{2A}) = u \text{ so } h_{1A},h_{2A}$ is <u>another way</u> of getting utility u

Therefore at prices p_{1B} , p_2 quantities h_{1B} , h_{2B} cannot cost more than h_{1A} , h_{2A} .

so in algebra $p_{1B}h_{1B} + p_2h_{2B} \le p_{1B}h_{1A} + p_2h_{2A}$

Remember the logic used here to derive the inequalities

Algebraic proof that the compensated demand curve cannot slope upwards

Inequalities from the two previous slides

 $p_{1A}h_{1A} + p_2h_{2A} \le p_{1A}h_{1B} + p_2h_{2B}$

 $p_{1\mathbf{B}}h_{1\mathbf{B}} + p_2h_{2\mathbf{B}} \le p_{1\mathbf{B}}h_{1\mathbf{A}} + p_2h_{2\mathbf{A}}$

Add the inequalities to get

 $p_{1\mathbf{A}}h_{1\mathbf{A}} + p_2h_{2\mathbf{A}} + p_{1\mathbf{B}}h_{1\mathbf{B}} + p_2h_{2\mathbf{B}}$

 $\leq p_{1A}h_{1B} + p_{2}h_{2B} + p_{1B}h_{1A} + p_{2}h_{2A}$

Remember the logic used here and derive the inequalities Everything that follows comes from simplifying this inequality. Simplify the inequality from the previous slides

$$p_{1A}h_{1A} + p_{2}h_{2A} + p_{1B}h_{1B} + p_{2}h_{2B}$$

$$\leq p_{1A}h_{1B} + p_{2}h_{2B} + p_{1B}h_{1A} + p_{2}h_{2A}$$

Subtract p_2h_{2A} and p_2h_{2B} from both sides to get

 $p_{1A}h_{1A} + p_{1B}h_{1B} \leq p_{1A}h_{1B} + p_{1B}h_{1A}$

Remember the logic used here and derive the inequalities Everything that follows comes from simplifying this inequality. From the previous slide

 $p_{1A}h_{1A} + p_{1B}h_{1B} \leq p_{1A}h_{1B} + p_{1B}h_{1A}$

Rearrange to get $0 \le (p_{1B} - p_{1A}) (h_{1A} - h_{1B})$

so if the price rises from p_{1A} to p_{1B} so $p_{1B} - p_{1A} > 0$

either $h_{1A} - h_{1B} = 0$ so compensated demand does not change

or $h_{1A} - h_{1B} > 0$ so compensated demand falls.

Compensated demand and the expenditure function with Cobb-Douglas utility 6. Compensated demand and the expenditure function with Cobb-Douglas utility

Step 1: What problem are you solving?

The problem is *minimising expenditure* $p_1x_1 + p_2x_2$ subject to

non-negativity constraints $x_1 \ge 0$ $x_2 \ge 0$

and the utility constraint $u(x_1, x_2) = x_1^{2/5} x_2^{3/5} \ge u$.

Step 2: What is the solution a function of?

Compensated demand is a function of prices & utility so is

 $h_1(p_1,p_2,u) = h_2(p_1,p_2,u)$

Step 3: Check for nonsatiation and convexity

We have already done this, both are satisfied.

Why does this matter?

See the next slide.

Essential logic

With nonsatiation and convexity



if there is a tangency point such as A

where MRS = p_1/p_2

and utility is u

this is compensated demand

because any cheaper point such as B gives lower utility.

Step 4: Use the tangency and utility conditions

Tangency requires that $MRS = p_1$

here we have already found

$$\frac{\frac{\partial \mathbf{u}}{\partial \mathbf{x}_{1}}}{\frac{\partial \mathbf{u}}{\partial \mathbf{x}_{2}}} = \frac{\frac{2}{5} \mathbf{x}_{1}^{-3/5} \mathbf{x}_{2}^{3/5}}{\frac{3}{5} \mathbf{x}_{1}^{2/5} \mathbf{x}_{2}^{-2/5}} = \frac{2\mathbf{x}_{2}}{3\mathbf{x}_{1}}$$

 p_2

Step 4: Use the tangency and utility conditions



because if $x_1^{2/5} x_2^{3/5} > u$ there is a cheaper way of getting utility u or more.

Step 5: Draw a diagram based on the tangency and utility conditions



Step 6: Remind yourself what you are finding and what it depends on.

You are finding *compensated demand* h_1 *and* h_2 which are functions of p_1 , p_2 and u.

Step 7: Write down the equations to be solved.

The equations are $x_1^{2/5} x_2^{3/5} = u$ and $\frac{2x_2}{3x_1} = \frac{p_1}{p_2}$

Step 8 solve the equations and write down the solution as a function.

This gives
$$x_1 = \left(\frac{2p_2}{3p_1}\right)^{3/5} u \quad x_2 = \left(\frac{3p_1}{2p_2}\right)^{2/5} u$$

usingnotation $h_1(p_1, p_2, u) = h_2(p_1, p_2, u)$ for compensated demand

$$h_1(p_1, p_2, u) = \left(\frac{2p_2}{3p_1}\right)^{3/5} u \qquad h_2(p_1, p_2, u) = \left(\frac{3p_1}{2p_2}\right)^{2/5} u$$

 $h_2(p_1, p_2, u) > 0.$

Note $h_1(p_1, p_2, u) > 0$,

Uncompensated demand

$$x_1(p_1, p_2, m) = \frac{2}{5} \frac{m}{p_1} \qquad x_2(p_1, p_2, m) = \frac{3}{5} \frac{m}{p_2}$$

Compensated demand

$$h_1(p_1, p_2, u) = \left(\frac{2p_2}{3p_1}\right)^{3/5} u \qquad h_2(p_1, p_2, u) = \left(\frac{3p_1}{2p_2}\right)^{2/5} u.$$

The expenditure function with Cobb-Douglas utility

Compensated demand is

$$h_1(p_1, p_2, u) = \left(\frac{2p_2}{3p_1}\right)^{3/5} u \qquad h_2(p_1, p_2, u) = \left(\frac{3p_1}{2p_2}\right)^{2/5} u$$

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so the expenditure function is

 $E(p_1, p_2, u) = p_1 h_1(p_1, p_2, u) + p_2 h_2(p_1, p_2, u)$

$$= p_1 \left(\frac{2p_2}{3p_1}\right)^{3/5} u + p_2 \left(\frac{3p_1}{2p_2}\right)^{2/5} u$$
$$= p_1^{2/5} p_2^{3/5} \left(\left(\frac{2}{3}\right)^{3/5} + \left(\frac{3}{2}\right)^{2/5}\right) u$$

Check

Compensated demand is function of prices and utility.

It is homogeneous of degree 0 in prices.

The expenditure function is a function of prices and utility.

It is homogeneous of degree 1 in prices.



The price of good 1 increases from p_1 to p_1 '.

Income and the price of good 2 do not change.

Demand moves from A to C.

 X_1



Substitution effect A to B,

Keep utility and p₂ constant

 X_1

Stay on the same indifference curve but move to a different tangency point because the price ratio changes.







0 m/p₁'

X₂

 $m/p_1 x_1$





The price of good 1 increases from p_1 to p_1' . Substitution effect A to B decreases X₁ increases X_2 . Income effect B to C decreases



X₂

The price of good 1 increases from p_1 to p_1' . Substitution effect A to B decreases X₁ increases X_2 . Income effect B to C decreases both x_1 and x_2 .

 X_1

Uncompensated demand

$$x_1(p_1, p_2, m) = \frac{2}{5} \frac{m}{p_1} \qquad x_2(p_1, p_2, m) = \frac{3}{5} \frac{m}{p_2}$$

Compensated demand

$$h_1(p_1, p_2, u) = \left(\frac{2p_2}{3p_1}\right)^{3/5} u \qquad h_2(p_1, p_2, u) = \left(\frac{3p_1}{2p_2}\right)^{2/5} u.$$

income effect: change in m on uncompensated demand

Uncompensated demand

$$x_1(p_1, p_2, m) = \frac{2m}{5p_1}$$
 $x_2(p_1, p_2, m) = \frac{3m}{5p_2}$

Compensated demand

$$h_1(p_1, p_2, u) = \left(\frac{2p_2}{3p_1}\right)^{3/5} u \qquad h_2(p_1, p_2, u) = \left(\frac{3p_1}{2p_2}\right)^{2/5} u.$$

income effect: change in m on uncompensated demand

Uncompensated demand

$$x_1(p_1, p_2, m) = \frac{2m}{5p_1}$$
 $x_2(p_1, p_2, m) = \frac{3m}{5p_2}$

Compensated demand

$$h_1(p_1, p_2, u) = \left(\frac{2p_2}{p_1}\right)^{3/5} u \qquad h_2(p_1, p_2, u) = \left(\frac{3p_1}{2p_2}\right)^{2/5} u.$$

substitution effect: change on p_1 on compensated demand



substitution effect $\frac{\partial h_1}{\partial p_1} =$



income effect
$$\frac{\partial x_1}{\partial m} = \frac{2}{5} \frac{1}{p_1} > 0$$

substitution effect $\frac{\partial h_1}{\partial p_1} = -\left(\frac{3}{5}\right)\left(\frac{2p_2}{3}\right)^{3/5} p_1^{-8/5} u < 0$

Compensated demand and the expenditure function with perfect complements utility

Compensated demand and the expenditure function with perfect complements utility


Finding compensated demand with perfect complements utility

 $u(x_1, x_2) = min(\frac{1}{2} x_1, x_2) = u$



Finding compensated demand with perfect complements utility

min($\frac{1}{2} x_1, x_2$) = u **X**₂ $X_2 = \frac{1}{2} X_1$ $\mathbf{\Omega}$ **X**₁

 $u(x_1, x_2) = min(\frac{1}{2} x_1, x_2) = u$

Expenditure minimization implies that (x_1, x_2) lies at the corner of the indifference curves so

 $X_2 = \frac{1}{2} X_1$

and gives utility u so

$$\frac{1}{2} X_1 = X_2 = U.$$

 $h_1(p_1,p_2,u) = 2u, h_2(p_1,p_2,u) = u.$

Uncompensated and compensated demand with perfect complements utility

uncompensated demand

$$x_1(p_1, p_2, m) = \frac{2m}{2p_1 + p_2}, \quad x_2(p_1, p_2, m) = \frac{m}{2p_1 + p_2}$$

 $h_1(p_1, p_2, u) = 2u$ $h_2(p_1, p_2, u) = u$ compensated demand

income effect



substitution effect

income effect
$$\frac{\partial x_1}{\partial m} = \frac{2}{2p_1 + p_2} > 0$$

^1

substitution effect
$$\frac{\partial h_1}{\partial p_1} = 0$$

X₂ m/p₁' $m/p_1 x_1$ 0

 $h_1(p_1,p_2,u) = 2u, h_2(p_1,p_2,u) = u.$



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 $h_1(p_1,p_2,u) = 2u, h_2(p_1,p_2,u) = u.$

The price of good 1 increases from p_1 to p_1 '.

There is <u>no substitution</u> <u>effect.</u>

The income effect reduces demand for x_1 and x_2 .

The expenditure function with perfect complements utility

Compensated demand is $h_1(p_1, p_2, u) = 2u$ $h_2(p_1, p_2, u) = u$ so the expenditure function is $E(p_1, p_2, u) = p_1 h_1(p_1, p_2, u) + p_2 h_2(p_1, p_2, u)$ $= p_1 2u + p_2 u = (2p_1 + p_2)u$

Properties of the expenditure function

8. Properties of the Expenditure Function

- 1. Increasing in utility
- 2. The expenditure function increases or does not change when a price increases.
- 3. Homogeneous of degree 1 in prices.
- 4. Shephard's lemma $\frac{\partial E(p_1, p_2, u)}{\partial p_1} = h_1(p_1, p_2, u)$

Definition of the expenditure function

The **expenditure function** E (p_1 , p_2 ,u) is the minimum amount of money you have to spend to get utility u with prices p_1 and p_2 . It is a function of p_1 , p_2 and u.

The amount of goods which minimizes the cost of getting utility u is compensated demand

so $E(p_1, p_2, u) = p_1h_1(p_1, p_2, u) + p_2h_2(p_1, p_2, u)$.

1. The expenditure function is increasing in utility



Utility increases from u_1 to u_2 .

The expenditure function increases.

2. The expenditure function increases or does not change when prices increase



The price of good 1 increases from p_{1A} to p_{1B} , compensated demand moves from A to B.

The expenditure function increases.

The expenditure function does not change if demand for good 1 is 0 at A.

3: The expenditure function is homogeneous of degree 1 in prices.

Compensated demand is homogeneous of degree 0 in prices.

If
$$k > 0$$
 $h_1(kp_1, kp_2, u) = h_1(p_1, p_2, u)$.

Expenditure function is homogeneous of degree 1 in prices.

If
$$k > 0$$
 $E(kp_1, kp_2, u) = k E(p_1, p_2, u)$.

Already explained.

Properties of the expenditure function 4 Shephard's Lemma $\partial E(p_1, p_2, u)$ $= h_1(p_1, p_2, u)$ ∂p.



 p_1

 p_{1A}

• By definition

compensated demand $h_1(p_1,p_2,u)$, $h_2(p_1,p_2,u)$

is the cheapest way of getting utility u at prices $p_1,\,p_2$

and costs $p_1 h_1(p_1,p_2,u) + p_2 h_2(p_1,p_2,u) = E(p_1,p_2,u)$

- Therefore any other way of getting utility u costs the same or more at prices p₁, p₂
- Thus if $u(x_1, x_2) = u$ then $E(p_1, p_2, u) \le p_1 x_1 + p_2 x_2$

 $h_{1A}(p_{1A},p_2,u)$ $h_{2A}(p_{1A},p_2,u)$ compensated demand with prices p_{1A},p_2 and utility u.

Therefore $u(h_{1A}, h_{2A}) = u$ and

$$p_{1A}h_{1A} + p_2 h_{2A} = E(p_{1A}, p_2, u)$$

 $h_1(p_1,p_2,u)$ $h_2(p_1,p_2,u)$ compensated demand with

prices p_1 , p_2 and utility u.

Therefore $u(h_1,h_2) = u$

and $p_1h_1(p_1,p_2,u) + p_2h_2(p_1,p_2,u) = E(p_1,p_2,u)$

 $E(p_1,p_2,u)$ is the cheapest way of getting utility u at prices (p_1,p_2)

 (h_{1A},h_{2A}) is another way of getting utility u, and

at prices (p_1,p_2) costs $p_1h_{1A} + p_2h_{2A}$ so

 $E(p_1, p_2, u) \le p_1 h_{1A} + p_2 h_{2A}$ for all p_1

 $\mathsf{E}(\mathsf{p}_{1\mathsf{A}},\mathsf{p}_{2},\mathsf{u}) = \mathsf{p}_{1\mathsf{A}}\mathsf{h}_{1\mathsf{A}} + \mathsf{p}_{2} \mathsf{h}_{2\mathsf{A}}$

New diagram

In this diagram (h_{1A}, h_{2A}) u and p_2 do not vary, p_1 varies.

Cost of buying (h_{1A}, h_{2A}) at prices (p_1, p_2) is $p_1h_{1A} + p_2h_{2A}$



$$\begin{split} \mathsf{E}(\mathsf{p}_{1},\mathsf{p}_{2},\mathsf{u}) &\leq \mathsf{p}_{1}\mathsf{h}_{1\mathsf{A}} + \mathsf{p}_{2} \mathsf{h}_{2\mathsf{A}} \quad \text{for all } \mathsf{p}_{1} \\ \mathsf{E}(\mathsf{p}_{1\mathsf{A}},\mathsf{p}_{2},\mathsf{u}) &= \mathsf{p}_{1\mathsf{A}}\mathsf{h}_{1\mathsf{A}} + \mathsf{p}_{2} \mathsf{h}_{2\mathsf{A}}. \end{split}$$

The graph of $E(p_1, p_2, u)$ cannot be anywhere inside the shaded area.



The graph of $E(p_1, p_2, u)$ meets the line with slope

 h_{1A} at A and is never above the line.

so the line is tangent to the graph of $E(p_1, p_2, u)$ at A

so the derivative of $E(p_1,p_2,u)$ with respect to p_1 at A is h_{1A} (compensated demand for good 1)



Shepard's Lemma

The derivative of the expenditure function

 $E(p_1, p_2, u)$ with respect to p_1

is compensated demand for good 1

$$\frac{\partial \mathsf{E}(\mathsf{p}_1,\mathsf{p}_2,\mathsf{u})}{\partial \mathsf{p}_1} = \mathsf{h}_1(\mathsf{p}_1,\mathsf{p}_2,\mathsf{u})$$

Try with previous examples

The Slutsky equation

9. The Slutsky equation

When $m = E(p_1, p_2, u)$

 $\frac{\partial x_1(p_1, p_2, m)}{\partial p_1}$

total effect of change in price on demand at constant income

$$= \frac{\partial h_1(p_1, p_2, u)}{\partial p_1} - \frac{\partial x_1(p_1, p_2, m)}{\partial m} x_1(p_1, p_2, m)$$

substitution effect of change in price on demand, at constant utility income effect of change in income on demand

Why bother with the Slutsky Equation?

It is the only way of knowing how big income and substitution effects are.

Combined with elasticity estimates it tells us that income effects are too small to bother with except for goods that are a large proportion of the budget.



A to B, change in <u>compensated</u> demand. This is the **substitution effect**.

B to C **income effect**. This is the shift in <u>uncompensated</u> <u>demand</u> when income m changes.

Intuition for the Slutsky Equation

When p_1 rises to $p_1 + \Delta p_1$ this is as if income falls by $x_1 \Delta p_1$ because this is how much more it costs to buy x_1 , so the effective change in income is $\Delta m = -x_1 \Delta p_1$.

The change in x_1 through the income effect is approximately

$$\frac{\partial x_1}{\partial m} \Delta m = -\frac{\partial x_1}{\partial m} x_1 \Delta p_1$$

The change in x_1 through the substitution effect is approximately



When
$$m = E(p_1, p_2, u)$$

$$\frac{\partial x_1(p_1, p_2, m)}{\partial p_1} = \frac{\partial h_1(x_1, x_2, u)}{\partial p_1} - \frac{\partial x_1(p_1, p_2, m)}{\partial m} x_1(p_1, p_2, m)$$

leave out the arguments to make this easier to write down

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial h_1}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1$$

multiply by $p_1 / x_1 = p_1 / h_1$ because $x_1 = h_1$
$$\frac{p_1}{x_1} \frac{\partial x_1}{\partial p_1} = \frac{p_1}{h_1} \frac{\partial h_1}{\partial p_1} - \frac{m}{x_1} \frac{\partial x_1}{\partial m} \frac{p_1 x_1}{m}$$

Slutsky equation in elasticities



But what are these elasticities?






Own price

elasticity of

uncompensated

demand





 $m \partial x_1$ $p_1 x_1$ $\frac{1}{x_1} \frac{1}{\partial m} \frac{1}{m}$



$\underline{p_1}$	∂h_1
h_1	∂p_1







Perloff notation

Income effects and the Slutsky Equation

 $\frac{p_1}{x_1}\frac{\partial x_1}{\partial p_1}$



Sign?



 ∂x_1 $p_1 x_1$ т Sign for a normal good Sign for an inferior good

Income effects and the Slutsky Equation $\frac{p_1}{h_1} \frac{\partial h_1}{\partial p_1}$ ∂X_1 CX_1 $p_1 x_1$ $\frac{1}{x_1} \frac{\partial p_1}{\partial p_1}$ m Sign for a normal Sign? good negative Sign for an inferior good

Income effects and the Slutsky Equation

 $\frac{p_1}{x_1}\frac{\partial x_1}{\partial p_1}$



Sign?

negative



Sign for a normal

good positive

Sign for an inferior



Income effects and the Slutsky Equation

 $\frac{p_1}{x_1}\frac{\partial x_1}{\partial p_1}$



Sign?

negative

 ∂x_1 $p_1 x_1$ т Sign for a normal good positive Sign for an inferior good negative.



Income effects are small for goods with a small budget share, even if the income elasticity of demand is quite large.

Quantitatively income effects often don't matter.



Which demand curve is more elastic A or B?

If good 1 is a normal good which is more elastic, compensated or uncompensated demand?

If good 1 is an inferior good which is more elastic, compensated or uncompensated demand?





Which demand curve is more elastic A or \underline{B} ?

If good 1 is a normal good which is more elastic, compensated or uncompensated demand?

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If good 1 is an inferior good which is more elastic, compensated or uncompensated demand?



Which demand curve is more elastic A or \underline{B} ?

If good 1 is a normal good which is more elastic, compensated or <u>uncompensated</u> demand?

If good 1 is an inferior good which is more elastic, <u>compensated</u> or uncompensated demand?

Expenditure minimization & compensated demand. What have we achieved?

- Income and substitution effects. Will be important for
 - labour supply, saving & borrowing.

- Downward sloping compensated demand
- Result from the Slutsky equation, income effect is small if budget share is small.
- Foundation for understanding welfare measures
 - consumer surplus, price indices, compensating and equivalent variation.